ELECTRICAL SIMULATION OF RADIANT HEAT-TRANSFER PROBLEMS

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The possibility of using electrical models in order to solve heat-conduction problems allowing for radiation is considered. Two methods are envisaged: that of nonlinear resistances and that of combined circuits. The latter is generalized to the case of a nonlinear problem.

Little attention has so far been devoted to the analysis of radiant heat-transfer problems, partly on account of the complexity of the processes taking place in radiation and partly on account of the mathematical difficulties arising in any attempts at solving the problem analytically.

Numerical methods of solving problems of radiant heat transfer are already well known; so are electrical-simulation techniques based on the principle of successive approximations [1].

The problem was solved in [2] by using continuously acting electronic machines, the equations originally expressed in partial derivatives being converted, for this purpose, into ordinary differential equations, using the method of straight lines. In this method the derivatives are approximately replaced by algebraical polynomials.

The devices which we shall here be describing enable us to solve the problem of radiant heat transfer without engaging in any iterations, the complicated temperature dependences of the thermal flux associated with the surfaces taking part in heat transfer being incorporated into the model by means of certain elements forming part of the actual apparatus.

It is well known that the expression for the amount of heat transferred by a body at a temperature T_1 to one at a temperature T_2 by radiation may be written as follows:

$$Q_r = k(T_1^4 - T_2^4). \tag{1}$$

If we remember that a direct solution of the heat-conduction equations may be achieved by using passive models (R and RC networks, electronic integrating circuits, etc.), then, as we shall subsequently show, radiant heat transfer may be simulated by using special pieces of equipment based on one of two principles: the method of nonlinear resistances [3] and the method of combined circuits.

It is hard to define the precise boundary between these two methods, since the second also incorporates the ideas of the method of nonlinear resistances; their practical embodiment takes the form of the construction of hybrid models comprising passive systems (R and RC networks) together with devices operating on the principle of electronic modeling. In a number of arrangements (including the one about to be described), nonlinear resistances are therefore accompanied by elements borrowed from electronic simulators.

The apparatus based on the first principle (Fig. 1) consists of four nonlinear resistances NR and also two summing devices SUM and a device giving multiplication by a constant factor MU based on dc amplifiers DCA. As nonlinear elements with volt–ampere characteristics of the $I = AU^4$ type, certain electronic tubes with an adjustable grid bias and a resistance set in parallel to regulate the slope of the characteristics may be employed. A study of their volt–ampere characteristics showed that the desired relationship could be obtained with triodes (and also certain pentodes) on using the initial parts of the characteristics.

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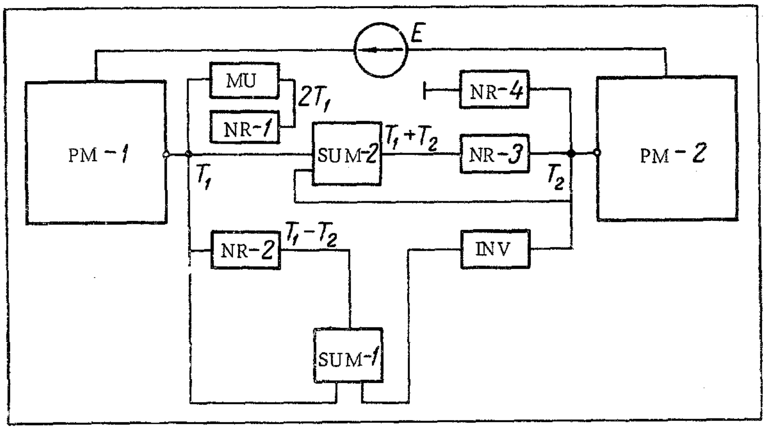

Fig. 1. Representation of boundary conditions using nonlinear resistances.

The nonlinear resistances NR-1 and NR-2 serve to represent Eq. (1) in the model of the radiating body; for this purpose the cathode of NR-1 and the anode (plate) of NR-2 are connected to the boundary point. The multiplier MU connected between the boundary point and the anode of NR-1 acts as a doubler. In this way a potential difference proportional to the temperature T_1 is created in the nonlinear resistance NR-1, and this means that a current proportional to T_1^4 flows through it.

The cathode of NR-2 receives a signal from the summing device (adder) SUM-1, in which the potential at the boundary point of the radiation body is summed with the inverted potential at the boundary point of the body receiving the radiation (inverted in the inverter INV). In this way the current flowing through NS-2 is proportional to T_2^4.

The nonlinear resistances NR-3 and NR-4 simulate Eq. (1) in the model of the body receiving the radiation. The first of these is connected by its cathode to the boundary point and by its anode to the summing device SUM-2, in which the potentials of the boundary points of the two bodies are added. The anode of the nonlinear resistance NR-4 is connected to the boundary point and the cathode to the zero busbar.

If we use the method of combined circuits, then there will be no nonlinear resistances (as such) in the model; Eq. (1) will be represented by circuits based on electronic simulation elements.

The apparatus (Fig. 2)* consists of a variable resistance R connected between units of the passive models PM-1 and PM-2. The boundary points of the models are connected to the inputs of the functional converters FC-1 and FC-2, in which the input signals are raised to the fourth power. The functional converters are connected to the adder–subtractor AS, which is also connected to the output of the amplifier A-1. The input of the amplifier A-1 receives a voltage taken from the measuring resistance R_m in series with R.

The apparatus operates on the principle of a tracking (servo) system. The resistance R is regulated by means of the servo drive motor D, the rotor of which is coupled mechanically to the slide of the resistance R, until the current in the circuit between the models PM-1 and PM-2 is proportional to the right-hand side of Eq. (1). The mismatch signal from the AS passes to the input of the servo drive amplifier A-2.

The method of combined circuits also constitutes the basis for other devices modeling radiant heat transfer. We shall consider one of these now. In contrast to the one just described, this apparatus does not belong to the class of servo systems; it has no mechanical couplings, but may be almost entirely constructed from devices existing in present-day passive models.

This system (Fig. 3a) incorporates two functional converters FC-1 and FC-2 raising the potentials of the boundary points to the fourth power, a summing device SUM, and two current stabilizers CS-1 and CS-2, the latter being formed, for example, by the channels representing boundary conditions of the second kind "GU-2" incorporated in the universal network model "USM-2" described in [4]. Between the summing device and CS-1 is an invertor INV designed to produce a current of the opposite sign in CS-1. In this way currents of equal and opposite sign are created in the current stabilizers, these currents being proportional to the voltages applied to the inputs, i.e., proportional to the right-hand side of Eq. (1).

* This circuit was developed in conjunction with S. I. Chervonnyi.

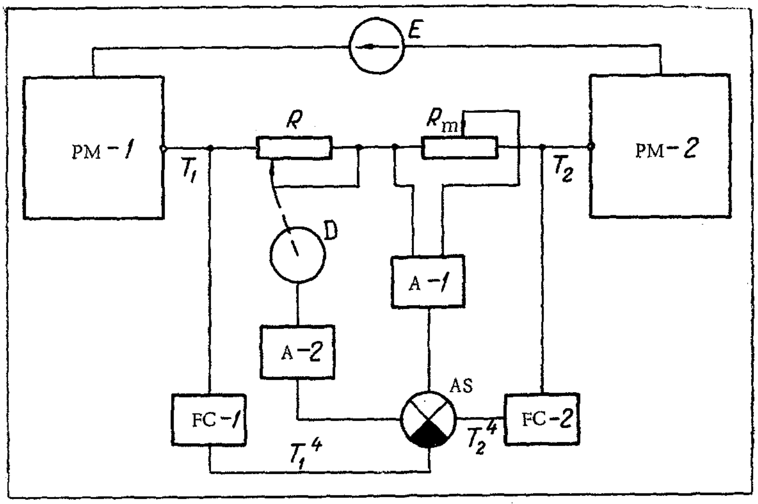

Fig. 2. Tracking (servo) system for modeling radiant heat transfer.

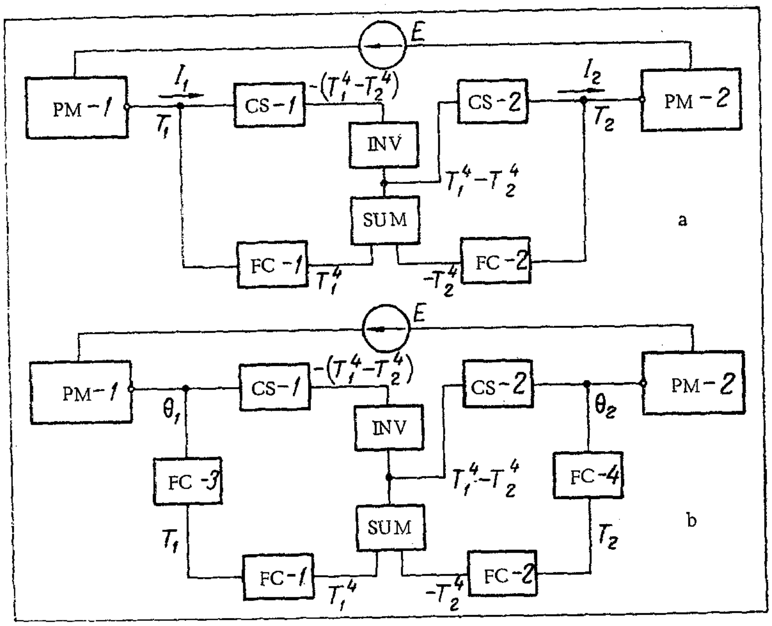

Fig. 3. Combined circuits for modeling radiant heat transfer; a) linear problem; b) nonlinear.

In solving a nonlinear problem of heat conduction with due allowance for radiation, we proceed in the same way as in [3, 5, 6].

Since the apparatus simulating radiant heat transfer may be used equally to solve steady-state and transient problems, we may for the sake of simplicity consider merely the problem of steady-state heat conduction.

The nonlinear equation of steady heat conduction

$$\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right]+\frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T}{\partial z}\right]=0 \tag{2}$$

may be reduced to the Laplace equation by means of several transformations. These include, for example, the integral Kirchhoff transformation

$$\theta=\int_0^T\lambda(T)\,dT. \tag{3}$$

In solving the nonlinear problem of radiant heat transfer between two bodies of different kinds, we require the introduction of two such functions as θ_1 and θ_2, the fields of these reflecting the potential fields obtained in the respective passive models.

Since the potentials of the boundary points of the models do not correspond to the true temperatures at these points, the use the device just described in the form illustrated in Fig. 3a is impermissible.

Figure 3b shows the block diagram of a system differing from the previous in that additional functional converters FC-3 and FC-4 are connected between the boundary points of the passive models and the corresponding converters FC-1 and FC-2; the new converters transform the potentials of the boundary points in proportion to the corresponding relationships $T = T(\theta_1)$ and $T = T(\theta_2)$ obtained by inverting expressions of type (3). The converters FC-3 and FC-4 may be based, for example, on dc amplifiers, with diode functional converters effecting transformation (3) in their feedback circuits.

A similar functional converter may be incorporated in the measuring circuit. This enables us to derive the temperature field directly rather than the field of the function θ, and avoids the necessity of a subsequent conversion from θ to the temperature.

It is not difficult to show that the solution of problems of radiant heat transfer using the devices here described offers certain advantages over existing numerical and analytical methods, since the problem is solved in a single process without any iterations. This is particularly important when solving a transient problem on models in which the process progresses continuously in time (RC network); without the devices in question the solution would be entirely impossible.

On the other hand, the method of straight lines and the associated approximate representation of the derivatives are not involved in this case.

The use of passive models for modeling the temperature fields of interacting bodies, however, has certain advantages over electronic models, which are most suitable for very simple problems.

NOTATION

T	is the temperature;
$k = \varepsilon_r C_r$;	
ε_r	is the reduced emissivity of the system;
C_r	is the reduced radiation coefficient;
λ	is the thermal conductivity;
x, y, z	are the Cartesian coordinates;
I	is the current;
U	is the voltage;
A	is the proportionality factor;
NR	is the nonlinear resistance;
SUM	is the summing device (adder);
MU	is the unit multiplication by a constant;
PM	is the passive model;
FC	is the functional converter;
AS	is the adder–subtractor;
A	is the amplifier;
R	is the resistance;
D	is the servo drive motor;
CS	is the current stabilizer;
INV	is the inverter;
E	is the supply source.

Subscripts

1, 2 refer to the radiating (emitting) body and to the body receiving the radiation, respectively.

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